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Josephson Junctions as Heterodyne Detectors

Y. TAUR, J. H. CLAASSEN, AND P. L. RICHARDS

***Abstract*—Heterodyne detection with a point-contact Josephson junction has been investigated both experimentally and theoretically. The measured performance of the device at 36 GHz is in good agreement with the theory. By operating vanadium point contacts at 1.4 K, we have achieved a single-sideband (SSB) mixer noise temperature of 54 K with a conversion gain of 1.35 and a signal bandwidth on the order of 1 GHz. From our results we can extrapolate a potentially impressive performance for these devices at submillimeter wavelengths.**

I. INTRODUCTION

JOSEPHSON junctions have long been considered promising as sensitive detectors of microwave and far-infrared radiation [1]. The principle of detection with Josephson junctions relies on the fact that the relation between the current and voltage for an ideal Josephson element [2] is highly nonlinear:

$$I(t) = I_c \sin\left[\frac{2e}{\hbar}\int_0^t V(t')\,dt'\right]. \quad (1)$$

An element satisfying this equation can be thought of as a lossless nonlinear inductor, which is generally useful for high-frequency device applications.

We have studied both experimentally and theoretically the use of point-contact Josephson junctions as heterodyne detectors with an externally applied local oscillator (LO). In this mode of operation a small signal is mixed with a relatively large LO signal in the junction to produce an output at a difference frequency much lower than the signal frequency. The experiments were performed with superconducting point contacts since they have a higher impedance and can be efficiently coupled to conventional waveguide modes more easily than other types of Josephson structures.

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II. THEORY AND SIMULATOR CALCULATIONS

It has been demonstrated that most of the properties of a point-contact Josephson junction can be understood in terms of the resistively shunted junction (RSJ) model [3], [4]. The model assumes that a real junction is equivalent to an ideal Josephson element defined by (1) in parallel with a constant resistor R. Although the product RI_c varies over a considerable range among different junctions, it has an upper limit given by the energy gap of the superconducting material from which the junction is made [5]. This product gives an intrinsic frequency $\omega_c = (2e/\hbar)RI_c$, which scales the important features of the RF response of the junction.

When a sufficiently large amplitude LO is applied to a Josephson junction, the zero-voltage current of the junction is suppressed and current steps appear on the static I–V curve of the junction at $V_{dc} = n\hbar\omega_{LO}/2e$, as shown in Fig. 1(a). The variation of the step heights with LO amplitude depends on the normalized frequency $\Omega = \omega_{LO}/\omega_c$. Fig. 1(b) and (c) shows the nature of such variations for the zeroth and first steps in the case $\Omega = 0.16$. If a small signal whose frequency ω_s is close to ω_{LO} is superimposed on the LO, the result is, in effect, an amplitude modulation of the LO at the intermediate frequency (IF) $|\omega_{IF}| = |\omega_s - \omega_{LO}| \ll \omega_{LO}$. We can therefore predict the IF output from a series of static I–V curves for various LO levels in the vicinity of that chosen to drive the junc-

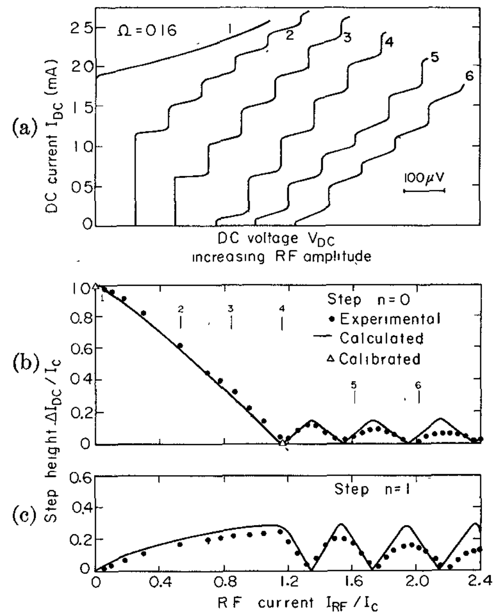

Fig. 1. Static I–V curve and step-height variations with RF amplitude.

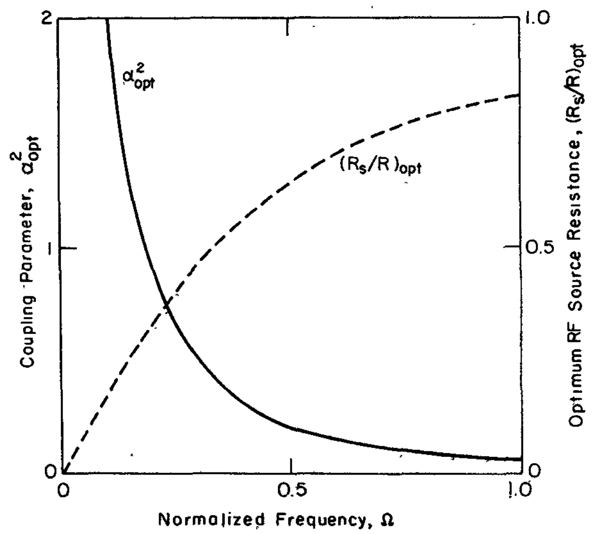

Fig. 2. Optimum coupling parameter and the corresponding RF source resistance computed from the RSJ model for broad-band coupling.

tion [6]. An expression for the conversion efficiency' defined as the ratio of the coupled IF power to the available signal power, has been derived to be [7]

$$\eta = C_{IF}(R_{dyn}/R)\alpha^2 \tag{2}$$

for bias voltages $V_{dc} \gg \hbar\omega_{IF}/2e$. Here C_{IF} is the output coupling efficiency, $R_{dyn} \equiv dV_{dc}/dI_{dc}$ is the dynamic resistance at the bias point, and α is a dimensionless parameter defined by

$$\alpha = \frac{\partial I_{dc}}{\partial[(8P_{LO}/R)^{1/2}]}. \tag{3}$$

Here P_{LO} is the available power from the LO source.

The maximum value of η usually occurs when the junction is dc biased between the zeroth and first steps. The parameter α is then proportional to the slope of the curve in Fig. 1(b) and is largest when the zero-voltage current is suppressed to about 0.1–0.5 of its value I_c in the absence of LO. Under these circumstances α^2 depends primarily on the normalized frequency Ω and the source resistance R_S of the LO. For the case of broad-band RF coupling, α^2 can be computed from previously published step-height calculations [7], [8]. In Fig. 2 we show the value of R_S/R which maximizes α^2, along with the resulting α_{opt}^2, as a function of Ω. For small Ω the coupling parameter α_{opt}^2 is large but the value of R_S required to achieve such large α^2 is considerably less than the junction shunt resistance R.

In practice the junction resistance R is much smaller than the waveguide characteristic impedance $Z_0 \approx 400\ \Omega$. Therefore, some impedance transformation is required to obtain efficient RF coupling to the junction. This was accomplished experimentally with a resonant structure which reduces the RF source resistance to $R_S \ll Z_0$ at frequencies close to the LO or signal frequency. The equivalent circuit shown in Fig. 3 for resonant coupling to a resistively shunted Josephson junction is very difficult to solve analytically. We have used an analog Josephson-junction simulator [9] to calculate α_{opt}^2 and $(R_S/R)_{opt}$ for the case of narrow-band resonant coupling. The results are similar to those shown in Fig. 2 for broad-band RF coupling, except when $R_S/R \ll 1$, which is very hard to realize experimentally.

With the junction simulator we have also investigated the low-frequency noise properties of a Josephson mixer assuming that the Johnson noise in the shunt resistor R is the only source of noise. At a bias point where the Josephson frequency is much larger than the noise frequency of interest, one can always define an equivalent noise temperature T_N of the junction as the available low-frequency noise power per bandwidth divided by the Boltzmann constant. We find it useful to present our noise results in terms of a dimensionless noise parameter

$$\beta^2 \equiv (T_N/T)(R/R_{dyn}) \tag{4}$$

where T is the physical temperature of the junction. Fig. 4 shows the calculated value of β^2 as a function of Ω for biasing conditions appropriate to heterodyne mixing. The zero-voltage current of the junction is suppressed by the LO to $I_c/2$, and the bias voltage is midway between the zeroth and first steps. These curves were obtained with

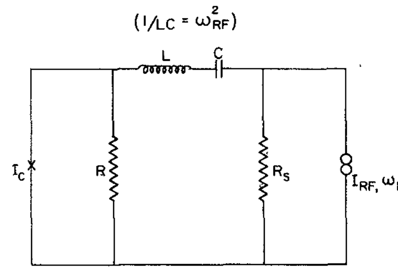

Fig. 3. Equivalent circuit for resonant RF coupling.

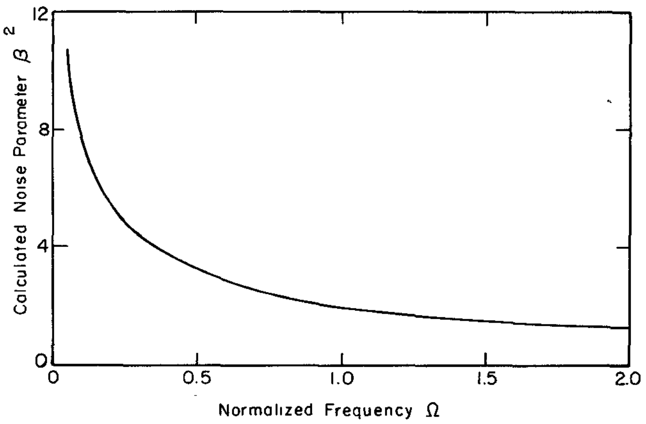

Fig. 4. Noise parameter β^2 obtained from the junction simulator under bias conditions appropriate to mixing.

the LO modeled as a current source driving the junction. At large normalized frequencies, the calculation predicts that β^2 approaches unity. However, at small Ω, β^2 can be appreciably larger than one, implying that various mixing processes in the junction have converted a large amount of high-frequency Johnson noise to low-frequency noise.

We have also calculated β^2 for the case of a resonantly coupled LO (Fig. 3). The result indicates that in general β^2 is larger than what is shown in Fig. 4 by a factor of 1.5–2.5. This excess noise arising from resonance is more pronounced for small normalized frequencies.

A convenient way to specify the performance of a mixer is to define a single-sideband (SSB) mixer noise temperature T_M by referring the intrinsic noise of the mixing element to the input of the mixer. In our case, T_M is given by

$$T_M \equiv C_{\mathrm{IF}} T_N/\eta = (\beta^2/\alpha^2) T. \quad (5)$$

For a given operating temperature, T_M is not very sensitive to the normalized frequency if $\Omega \lesssim 1$ according to our calculations shown in Figs. 2 and 4. However, the importance of noise in the IF amplifier is determined by the conversion efficiency, which does depend strongly on Ω.

III. EXPERIMENTAL RESULTS AT 36 GHZ

In order to check these theoretical ideas, we have set out to obtain a quantitative experimental understanding of a Josephson-effect mixer at a signal frequency of 36 GHz. The apparatus shown in Fig. 5 has been used for our experiments. The point contact is made along the E-field direction in the middle of the standard A-band waveguide. The microwave radiation is resonantly coupled to the junction by means of a choke plunger and a tuning stub $3\lambda_g/4$ in front of the junction. We have used a choke flange to electrically insulate one side of the junction so that the microwave leakage along the superconducting wire is negligible compared with the junction loss. The signal and LO power were supplied by two klystrons with frequencies set 50 MHz apart. A calibrated cold attenuator

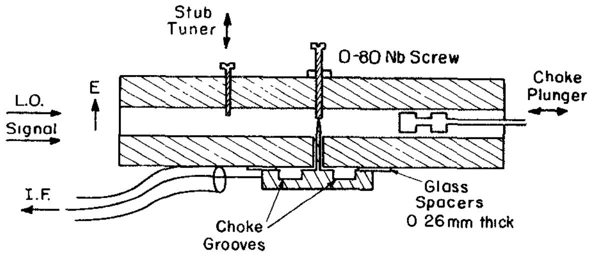

Fig. 5. Niobium waveguide assembly for a 36-GHz Josephson-junction mixer.

is placed in the waveguide in front of the stub to isolate the junction from room-temperature blackbody radiation. The IF output at 50 MHz from the junction is coupled via a cryogenic coaxial cable to an Avantek-UA142 amplifier whose noise temperature is 110 K when matched to a 50-Ω source.

We have used niobium and vanadium to make point-contact Josephson junctions at 8, 4.2, and 1.4 K. When a junction with a suitable I–V characteristic is obtained, we adjust the plunger to maximize the variation of the zero-voltage current for a given change in the LO power. Then we measure the coupling parameter α^2 from a series of I–V curves measured at different LO levels. Some representative data are given in Table I, where it is found that for junctions with $R \gtrsim 15\ \Omega$, the measured value of α^2 is between 0.8 and 1.0 times α_{opt}^2 calculated for the same normalized frequency (Fig. 2). This shows that the plunger and stub provide the proper impedance transformation for optimum coupling to these junctions.

For the well-coupled junctions in Table I, we measured the SSB small-signal conversion efficiency when the LO level was set such that the zero-voltage current of the junction was reduced to approximately $I_c/2$. This was done by using a spectrum analyzer and a calibrated source to determine the amount of IF power at 50 MHz coupled to the amplifier. As expected, η is largest when the junction is biased midway between the zeroth and first steps where R_{dyn} reaches a maximum. These peak values are given in Table I and are found to agree with (2) within ±20

TABLE I
SUMMARY OF MIXING RESULTS AT 36 GHz

Material	T(K)	R(ohm)	Ω	α^2	R_{dyn}/R	η	T_N(K)	β^2	T_M(K)
Nb	8	28	0.48	0.22	2.1	0.34	80	4.7	210
Nb	8	5	0.50	0.18	30	4.05	1500	6.3	260
Nb	4.2	19	0.21	0.74	0.8	0.47	76	21.5	120
Nb	4.2	16	0.36	0.27	2.3	0.55	85	8.8	140
V	1.4	17	0.33	0.34	2.4	0.66	40	12.1	55
V	1.4	25	0.30	0.36	4.4	1.35	97	15.7	54

percent. The fact that values of conversion efficiency greater than unity could be obtained in our 36-GHz experiment is of considerable practical interest. Also, we find that, unlike other frequency converters making use of a nonlinear reactance, the conversion efficiencies are observed to be independent of the sign and magnitude of $\omega_s - \omega_{LO}$.

The bandwidth of our mixer was determined directly, by varying ω_s; the observed value of ~500 MHz (both sidebands) appears to be limited by the IF coupling to the junction. The ultimate bandwidth limit is probably set by the Q of the RF coupling, which could be determined in our arrangement by measuring α^2 as a function of RF frequency for fixed positions of the tuning elements. For a 30-Ω junction, we found a coupling bandwidth of ~2 GHz. The dynamic range of a Josephson mixer is expected to be proportional to the product RI_c^2 of the junction. In our experiments, we found that the IF output saturates at an RF level approximately $0.01RI_c^2$; this number is typically on the order of 10^{-10} W.

The noise of our system is measured by using an HP-411A RF power meter to read the amplified noise within a bandwidth of 50 MHz ± 10 MHz determined by a bandpass filter. In order to measure T_N and T_M, we must calibrate the amplifier noise contribution to the total noise power. This was done by replacing the superconducting wires with copper wires to form a metal-to-metal point contact which we assume behaves as an ordinary resistor with Johnson noise. We measure the total noise power for a range of junction resistances at three temperatures: 300, 77, and 4.2 K. Since the Johnson noise power from the resistive point contact is proportional to its physical temperature T, we can determine the amount of amplifier noise as a function of the source resistance. These noise measurements also give the value of $C_{IF} = dP_N/d(kT)$, where P_N is the total IF noise power per bandwidth referred to the input of the amplifier. For IF source resistances within the range of 20–100 Ω, we found that C_{IF} is between 0.8 and 0.9.

Since the IF source resistance of a Josephson junction is equal to its dynamic resistance R_{dyn}, we can determine T_N for the junction by measuring the total noise power and R_{dyn} at the bias point. The junction noise temperatures measured under the same conditions from which η was obtained are included in Table I, along with the values of $\beta^2 \equiv (T_N/T)(R/R_{dyn})$. We found that the measured β^2 has similar dependence on Ω to that predicted by the simulator calculation in Fig. 4. However, the values of β^2 listed in Table I are larger by a factor of 2–4 depending on the normalized frequency than those calculated for the case of a constant-current LO source. We believe that this difference is at least partially due to the resonant coupling. An important consequence of the qualitative agreement between our measurements and theory is that the obtained T_M is roughly proportional to the junction temperature T. The best performance that we have achieved is a mixer noise temperature of 54 K with conversion gain (SSB) 1.35 for a vanadium junction operated at 1.4 K.

IV. DISCUSSION AND CONCLUSION

In this work we have shown good agreement between our experimental results at 36 GHz and the simulator calculations based on the RSJ model for RF coupling and IF noise in a Josephson-effect heterodyne detector. The noise temperature of our Josephson point-contact mixer is significantly lower than that of cooled Schottky-diode mixers [10]. The large values of conversion efficiency that we obtained have the effect of reducing the contribution of the IF amplifier noise to the total system noise. Furthermore, since only a small amount of LO power is required for optimum operation of a Josephson mixer (≈1 nW compared with ≈1 mW for Schottky-diode mixers), the problem of LO noise is eliminated and a balanced mixer is not required. This is especially important for heterodyne systems at higher signal frequencies where a coherent LO with large power is very difficult to construct.

We have demonstrated that the performance of a Josephson-effect mixer depends only on the ratio $\omega_s/\omega_c = \Omega$. The experimental results quoted in Table I for $\omega_s = 36$ GHz were obtained from junctions whose RI_c product was approximately 1/10 of the energy gap. Point-contact Josephson junctions with values of RI_c approaching the energy gap have been fabricated [11]. With such junctions, results similar to our 36-GHz mixer should be attainable at appropriately scaled signal frequencies.

An important parameter for these devices is the junction shunt resistance R. If R is too large, the critical current of the junction is usually very small and all the steps are severely rounded by noise. Empirically, we find that the conversion efficiency measured under these circumstances is less than the expected value, which suggests that the junction model we have used may not be valid in this limit. On the other hand, R cannot be too small, for otherwise it is very difficult to achieve good signal coupling to the junction. According to our experience at 36 GHz, these problems can be avoided for values of R in the range 10–40 Ω.

To be explicit about our extrapolation to higher frequencies, let us consider the case of a Josephson junction with $R = 20$ Ω and $I_c = 30$ μA operated as a 300-GHz mixer at 1.4 K. This RI_c product (≈1/4 of the energy gap of Nb) corresponds to $\Omega = 1.0$ for $\omega_s/2\pi = 300$ GHz and can be achieved even with our rather crude techniques for making point contacts. At such a large normalized frequency, the extra noise arising from resonant coupling is negligible. It can be seen from Figs. 2 and 4 that in this case $\alpha_{opt}^2 = 0.06$ and $\beta^2 = 2$. Since there exist practical problems in making a good-quality microwave-coupling circuit at such high frequencies, we assume that the experimental value of the coupling parameter is $\alpha^2 \approx 0.4\alpha_{opt}^2 \approx 0.025$. These rather conservative numbers predict a mixer noise temperature of approximately 120 K and a conversion efficiency slightly less than 0.1, which should represent a very useful device in this frequency range.

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Material Characterization and Ultimate Performance Calculations of Compensated n-Type Silicon Bolometer Detectors at Liquid-Helium Temperatures

C. J. SUMMERS AND S. ZWERDLING

***Abstract*—The dependence of the resistivity and far infrared (FIR) absorptance on donor concentration, compensation, and temperature in compensated n-type Si is reported. The effect of environment, time constant, and spectral passband on the noise equivalent power (NEP) of the compensated Si bolometer is examined and compared with similar calculations for the compensated Ge bolometer.**

INTRODUCTION

The important characteristics required for a semiconductor bolometer element with a low-noise equivalent power (NEP) are: 1) a high value for the temperature coefficient of resistance α_R, 2) a value of resistance R compatible with a low-noise preamplifier, 3) a high absorptance A_t, and 4) a low value for the thermal capacitance $\mathcal{C}$. Suitably doped Ge crystals partially fulfill all these requirements. However, the smaller thermal capacitance of Si and the larger temperature dependence of the resistivity reported for properly doped crystals indicate that Si may be superior to Ge as the impurity-host crystal for a far infrared (FIR) bolometer element. This paper reports on an investigation of this possibility and on the doping levels in Si which lead to optimum detector performance. The first requirement is best satisfied by a Si crystal having a small donor concentration N_D and a low compensation ratio K_n, whereas satisfying the second and third requirements necessitates a large N_D value and a reasonably large value of K_n. A series of samples with N_D in the range 0.5–2 $\times 10^{18}/\text{cm}^3$ and K_n between 0.05–0.4 was therefore prepared and characterized to determine those electrical and optical properties from which their performance as bolometer elements can be calculated.

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MATERIAL CHARACTERIZATION

Measurements to determine the dependence of the resistivity ρ on temperature T have been performed between 1.5–4.2 K and the data fitted to the expression $\rho = \rho_0 \exp(AT^{-n})$ using a least squares regression. The parameter ρ_0 is a constant and the quantity A is related to α_R through the expression: $\alpha_R = 1/R(dR/dT) = -nAT^{-(n+1)}$. Two recent theories of hopping conduction [1], [2] have concluded that $\ln \rho$ should have a $T^{-1/4}$ or